CHECK YOUR ANSWERS

1. SAS
2. ASA
3. HL
4. SSS
5. AAS
6. $x = 7$

Chapter 14

TRIANGLE BISECTORS

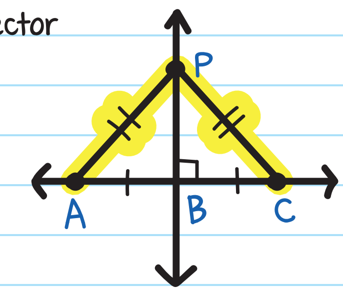
PERPENDICULAR BISECTORS

Perpendicular bisectors always cross a line segment at right angles (90°), cutting it into two equal parts.

PERPENDICULAR BISECTOR THEOREM

If a point is on the perpendicular bisector of a line segment, then the point is **EQUIDISTANT** to the segment's endpoints.

at equal distances



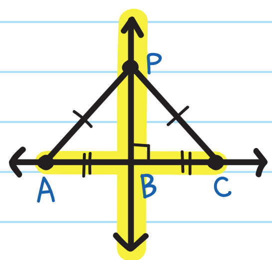
If point P is on the perpendicular bisector of $\overline{AC}$, then $AP = PC$.

The converse of this theorem is also true.

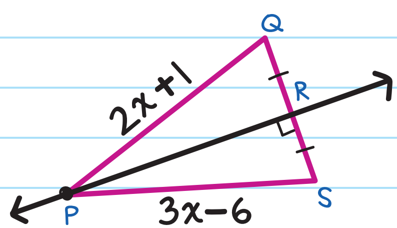
CONVERSE OF PERPENDICULAR BISECTOR THEOREM

If a point is equidistant to the endpoints of a segment, then it is on the perpendicular bisector of that segment.

If $AP = PC$, then point P is on the perpendicular bisector of $\overline{AC}$.



EXAMPLE: Find the value of x in the figure.



Since $\overleftrightarrow{PR}$ is a perpendicular bisector of $\overline{QS}$, P is equidistant to Q and S.

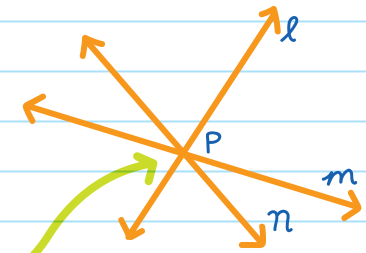
$PQ = PS$

$2x + 1 = 3x - 6$

$x = 7$

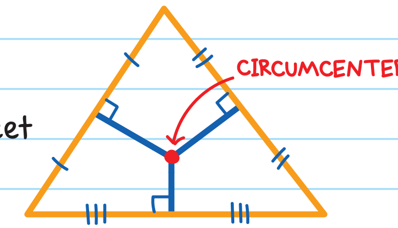
When three or more lines intersect at one point, they are **CONCURRENT**. Their point of intersection is called the **POINT OF CONCURRENCY**.

Lines ℓ, m, and n are concurrent. P is their point of concurrency.

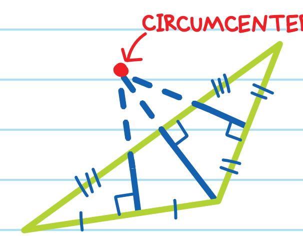


CIRCUMCENTER

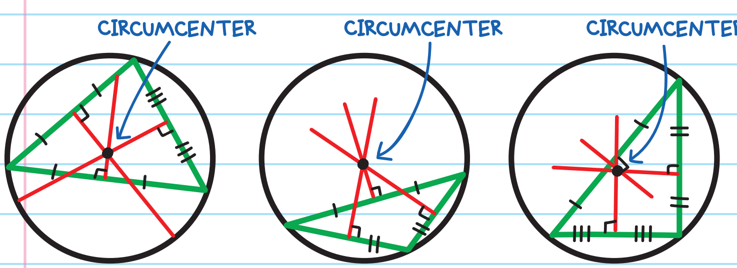
In a triangle, there are three perpendicular bisectors that all meet at one point, the **CIRCUMCENTER**.

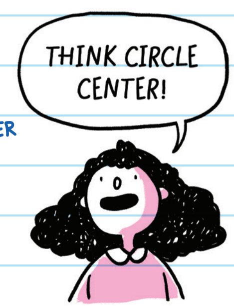


The circumcenter can be outside or inside the triangle.



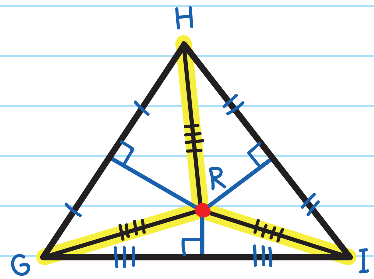
We can draw a circle through the three vertices of any triangle. The circumcenter of the triangle will be the center of the circle.





CIRCUMCENTER THEOREM

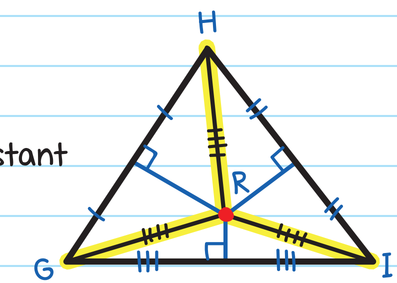
The circumcenter of a triangle is equidistant to the vertices.



If R is the circumcenter of $\triangle GHI$, then $HR = GR = RI$.

EXAMPLE: In $\triangle GHI$, $HR = 3x - 7$, $GR = x + 3$.

Find the value of RI.



Since the circumcenter is equidistant to the vertices, $HR = GR = RI$.

Step 1: Find the value of x.

$HR = GR$

$3x - 7 = x + 3$

$2x - 7 = 3$

$2x = 10$

$x = 5$

Step 2: Calculate HR (or GR—they are the same length).

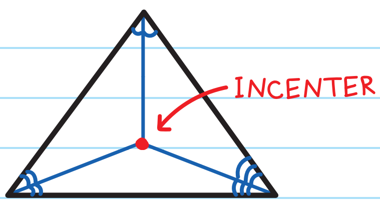
$HR = 3x - 7 = 3(5) - 7 = 8$

Since $HR = RI$,

$RI = 8$

INCENTER

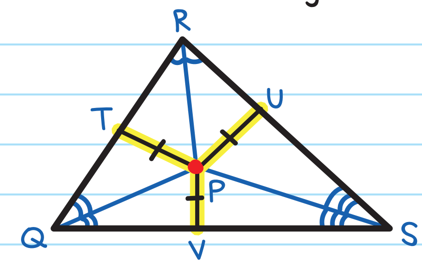
In a triangle, the angle bisectors of the three interior angles all meet at one point. This point is at the center of the triangle and is called the **INCENTER**.



INCENTER THEOREM

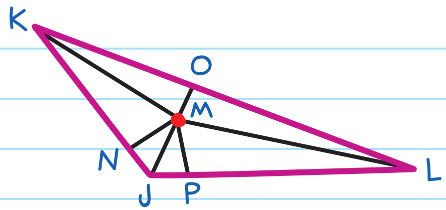
The incenter is equidistant to the sides of the triangle.

If P is the incenter, then $PT = PU = PV$.



EXAMPLE: If M is the incenter of $\triangle JKL$, $MN = 3x + 16$, and $MP = 7x + 12$, find MO.

From the incenter theorem,
$MN = MP = MO$.



Step 1: Find the value of x.

$MN = MP$

$3x + 16 = 7x + 12$

$16 = 4x + 12$

$4 = 4x$

$x = 1$

Step 2: Find the value of MO.

Substituting the value of x into MN,

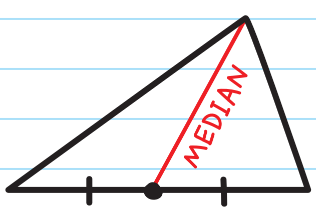
$MN = 3x + 16 = 3(1) + 16 = 19$

Since $MN = MO$,

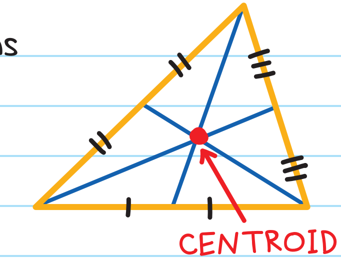
$MO = 19$

MEDIAN AND CENTROID

A **MEDIAN** of a triangle is a line from a vertex to the midpoint of the opposite side.



Every triangle has three medians which meet at a point called the **CENTROID**.

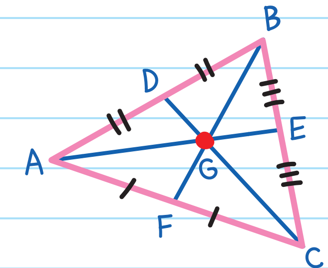


CENTROID THEOREM

The centroid is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

If G is the centroid of $\triangle ABC$, then

$BG = \frac{2}{3}BF$, $AG = \frac{2}{3}AE$, $CG = \frac{2}{3}CD$

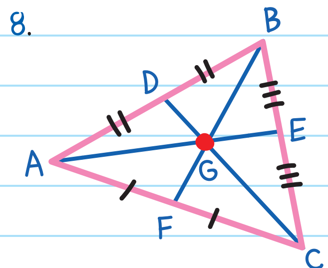


EXAMPLE: In $\triangle ABC$ above, $BG = 8$. Find the measures of GF and BF.

From the Centroid Theorem,

$BG = \frac{2}{3}BF$

$8 = \frac{2}{3}BF$



$8 \times 3 = \frac{2}{3} BF \times 3$ Multiply both sides by 3.

$24 = 2 \times BF$ Divide both sides by 2.

$BF = 12$

We can now find GF using the **SEGMENT ADDITION POSTULATE**:

$BF = BG + GF$

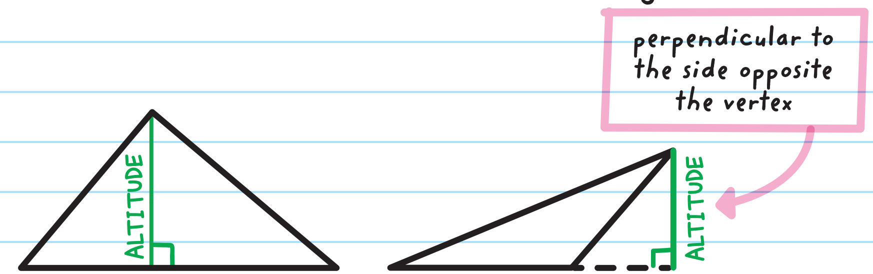
$12 = 8 + GF$

$GF = 4$

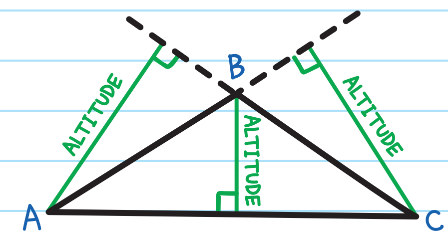
If you wanted to balance a triangle plate on one finger, you would need to place your finger on the centroid to balance it. This point is called the **center of gravity**—the point where the weight is equally balanced.

ALTITUDE AND ORTHOCENTER

The **ALTITUDE** of a triangle is the line segment from a vertex to the opposite side, and perpendicular to that side. An altitude can be outside or inside the triangle.

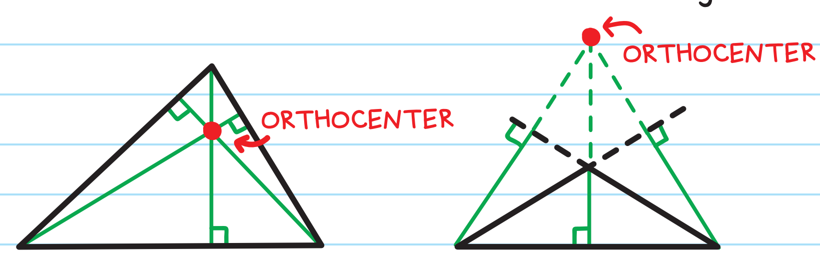


Every triangle has three altitudes.

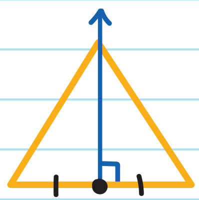
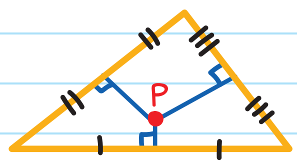
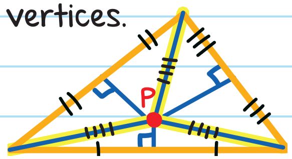
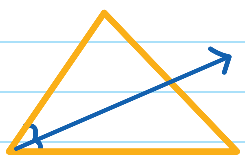
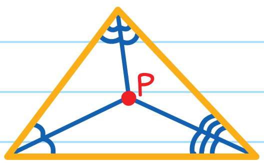
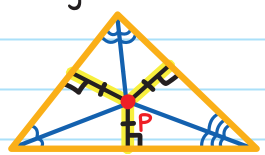
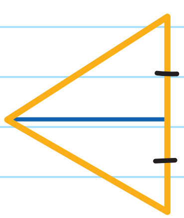
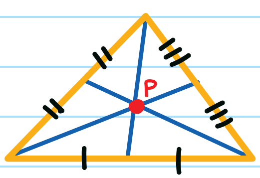
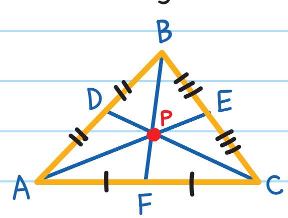
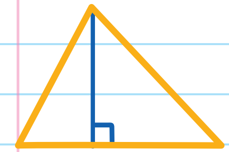
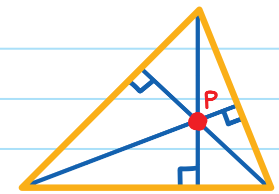


The point where the altitudes of a triangle meet is the **ORTHOCENTER**.

The orthocenter can be outside or inside the triangle.



Triangle bisectors and their points of concurrencies:

TERM	POINT OF CONCURRENCY (P)	THEOREM
perpendicular bisector 	circumcenter 	The circumcenter of a triangle is equidistant to the vertices. 
angle bisector 	incenter 	The incenter is equidistant to the sides of the triangle. 
median 	centroid 	If P is the centroid of $\triangle ABC$, then $BP = \frac{2}{3}BF$, $AP = \frac{2}{3}AE$, $CP = \frac{2}{3}CD$ 
altitude 	orthocenter 	No theorem for this one.

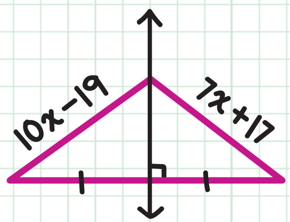
A way to help remember the term that matches each point of concurrency:

Median—Centroid, Altitude—Orthocenter, Perpendicular Bisector—Circumcenter, Angle Bisector—Incenter.

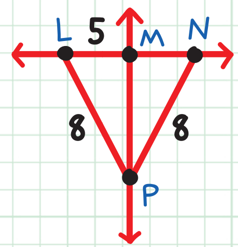
My cat ate old peanut butter cookies and became ill.

CHECK YOUR KNOWLEDGE

1. Find the value of *x*.

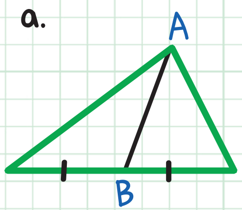


2. Find the measure of MN.

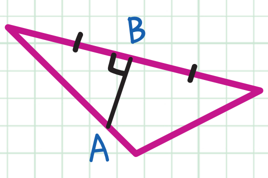


3. For triangles in illustrations a, b, and c below, state whether AB is a perpendicular bisector, median, or altitude.

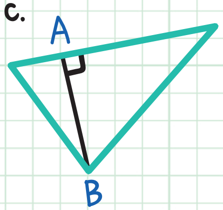
a.



b.

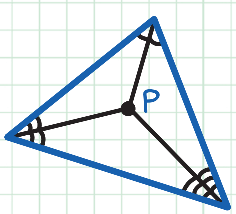


c.

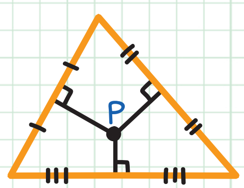


For questions 4–7, determine if point P is the incenter, circumcenter, centroid, or orthocenter of the triangle.

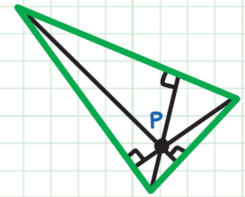
4.



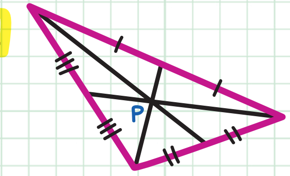
5.



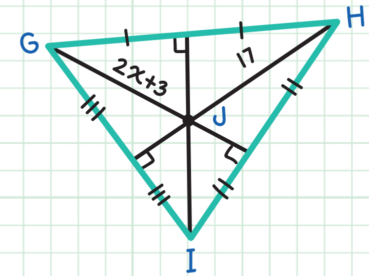
6.



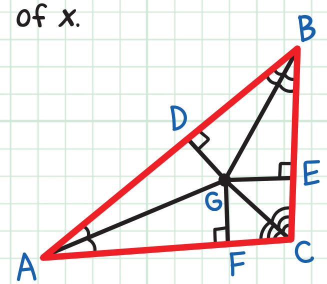
7.



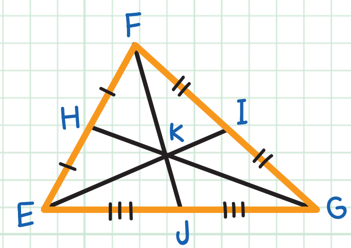
8. Find the measure of JI in △GHI below.



9. In △ABC, DG = 2*x* + 3 and GF = 3*x* − 7. Find the value of *x*.



10. In the triangle below, EI = 135. Find the measures of EK and KI.



ANSWERS

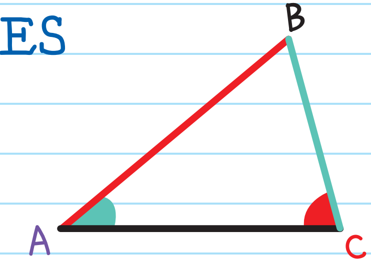
CHECK YOUR ANSWERS

1. $10x - 19 = 7x + 17$; therefore, $x = 12$
2. MN = 5
3. a. median; b. perpendicular bisector; c. altitude
4. incenter
5. circumcenter
6. orthocenter
7. centroid
8. JI = 17
9. $2x + 3 = 3x - 7$; therefore, $x = 10$
10. $EK = \frac{2}{3}(135)$; therefore, $EK = 90$, $KI = 45$

Chapter 15

TRIANGLE INEQUALITIES

COMPARING SIDES AND ANGLES



When comparing two sides of a triangle, the angle opposite the longer side is larger than the angle opposite the shorter side.

If $\overline{AB} > \overline{BC}$, then $m\angle C > m\angle A$.

When comparing two angles of a triangle, the side opposite the larger angle is longer than the side opposite the smaller angle.

If $m\angle C > m\angle A$, then $\overline{AB} > \overline{BC}$.